NON-LINEAR TARIFFS AS A VERTICAL CONTROL TOOL IN DISTRIBUTION CHANNELS WORK IN PROGRESS

by

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Abstract

In this paper, we evaluate the effectiveness of a contract implementing non-linear tariffs as a way to achieve perfect coordination in a supply chain. In the initial model, we face an inventory system in a distribution channel formed by a single wholesaler and a single retailer. The latter faces a price-sensitive demand at the final market, where we assume a linear pricing is applied. The wholesaler and the retailer act independently and maximize their own benefits under complete information. We show that when the wholesaler can establish a contract implementing non-linear tariffs with the retailer, we can reproduce the benefits of vertical integration, even when we are in the presence of a supply chain with inventory decisions. The strategy is not only simpler than alternatives approaches found in the literature (for example Viswanathan and Wang, 2002), but also it is easily extended to more general settings

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1 Introduction

The look for perfect coordination in a distribution chain has been a traditional challenge for researchers over the past decades. The reference has been the output obtained by a vertically integrated system where the different parts of the chain achieve, by definition, perfect coordination leading to maximum profits.

However, vertical integration is costly and it cannot be viable in some contexts. Consequently, perfect coordination should be pursued by alternative means based on the contractual arrangements among the participants in the chain. In the simplest setting, the literature has considered a structure composed of two vertically related firms, one wholesaler and one retailer, where production and inventory decisions must be made. For this particular and simple setting, many studies have been dedicated to study the contractual relation that can be employed to obtain profits as close as possible to those arising from perfect coordination, but in the absence of vertical integration.

Viswanathan and Wang (2002) studied the impact of the combined use of quantity and volume discounts as a contractual tool and its impact on profits. The results showed that profits from vertical integration can be matched, in some particular settings, by the use of carefully designed combination of discounts. Wang (2005) showed that a discount policy helps to align incentives for the firms and increase the feasibility to achieve coordination. Qina et al. (2007) focused on the joint use of volume discount and a franchise contract. They determined that perfect coordination, and then maximum profits, is also achievable using this tool.

On the other hand, Ching-chong et al. (2010) established conditions under which the level of profits under the vertically integrated regime will always be greater than under a vertically separated regime. Bonnano and Vickers (1987) introduced strategic interaction considerations, when two different firms have to decide between distributing their product using their own channels or through a retailer.

Toktas-Palut and Ülengin (2011) applied three different types of tools in the case of more than one retailer to achieve an improved coordination in the chain: contracts with subsidized inventory maintenance costs, transfer of payment contract based on the Pareto improvement and cost-sharing contracts. They concluded that although the three mechanisms established by the wholesaler permit to achieve perfect coordination, the cost-sharing contracts seem to be the preferred by the parties. Finally, Khouja (2003) and Abdelaziz et al. (2011) also studied the same kind of problem, but they focused on different aspects: the necessity to coordinate the cycle periods in the chain and the possibility to delay payments, respectively.

A different branch of the literature has focused on the algorithms developed to calculate inventory decisions and the best prices for the distribution chain. In this ine we have Roundy (1985), the variations introduced by Abdul-Jalbar et al. (2010) and the genetic algorithms developed by Cha et al. (2008). Monthatipkula and Yenradee (2008) developed an inventory control system for the chain which allows the total associated costs to be reduced.

The aim of this work is to use a different vertical control strategy that provides perfect coordination in a distribution chain with inventory decisions and price-sensitive demands. The tool is based on the use of nonlinear contracts, implemented by the wholesaler, that provides the retailer with the incentives to pursue the integrated benefits, which can extracted by the wholesaler as a fixed charge. The advantage of this approach is its simplicity and applicability in different settings.

The rest of the paper is organized as follows. In section 2 we establish the main assumptions and notation, in section 3 we provide the main results for the basic model while section 4 contains some significant extensions. Conclusions are relegated to section 5.

2 The Basic Model

In the initial model we consider the interaction of one wholesaler and one retailer, each one of them exerting monopoly power at its level in a distribution chain of one product for a specific period, standardized here to one year. We assume this is a dynamic game of complete information, so all the agents are aware of the benefits and costs faced by him/herself and the other agent in the chain. Moreover, each firm maximizes its own benefits. Figure 1 summarizes the relevant variables and interactions.

The wholesaler sells the product to the retailer at a price C, incurring in a cost C_v per unit. The retailer faces an inventory cost h, for each unit per year, while the seller faces an inventory cost h_0 per unit per year. Likewise, whenever the retailer places an order, he incurs a cost k, and imposes on the wholesaler a cost A to satisfy that order. The wholesaler also incurs in a cost k_0 for every order he places to its providers.

The retailer faces a price-sensitive deterministic demand $D(p)$ in each period and must choose a linear pricing. Calling $\psi(D)$ the inverse demand function ($p = \psi(D)$), one can write the retailer's income as $R(D) = \psi(D)D$. We assume that the inverse demand function is decreasing and weakly concave, and therefore the retailer's income will be concave in D for $D > 0$.

2.1 The profit functions

The costs incurred by the retailer start with the purchase of the product units within the period, and correspond to CD. The retailer must define the quantity q to request in each order, so that total ordering costs in the period will be given by kD/q. The total inventory costs in a period, by the usual argument, is given by $hq/2$.

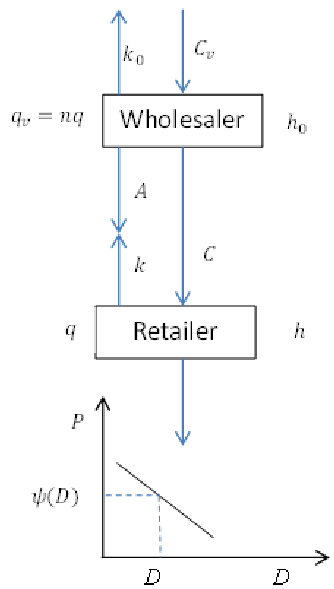

Figure 1: Diagram of the basic model

As a result, the retailer's profits are given by:

$$\pi_r(D,q) = R(D) - CD - h\frac{q}{2} - k\frac{D}{q}$$

Likewise, we can define the wholesaler profits. The revenues arise exclusively from selling the product to the retailer, so they are given by DC. On the other hand, the costs have several components. First, a direct cost of $C_v D$ by the D units bought. Second, it is well known that for a given level of units ordered by the retailer (q), the wholesaler place orders of size nq, where n is an entire positive number. As a consequence, the inventory held by the wholesaler in a period corresponds to what is shown in figure 2:

Therefore the average inventory held by the wholesaler is obtained as the area below the curve in Figure 2 which is given by $[(n-1)q]/2$. Moreover, the wholesaler incurs in a cost k_0 every time he places an order, and then the total order cost is given by $k_0 D/(nq)$ per year. Finally, the wholesaler incurs in a unitary cost A to satisfy a retailer's order, so total costs associated to this component is: AD/q.

As a result, the wholesaler's profits can be written as:

$$\pi_v(C,n) = CD - C_v D - h_0\frac{(n-1)q}{2} - k_0\frac{D}{nq} - A\frac{D}{q}$$

Note that these profits are only a function of n and C, as for a given C, the retailer will

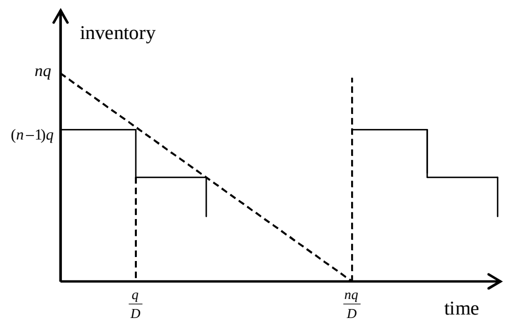

Figure 2: Units held by the wholesaler in inventory as a function of time

be who indeed defines the demand to be supplied and the size of the batch he will order. Then the wholesaler defines the price to be charged for the product and the corresponding size of his batch.

2.2 The integrated monopoly

The vertically integrated structure corresponds to the reference case to compare the benefits under any other structure. The profits in this case correspond to the simple sum of the individual profits of the companies. We are assuming here that costs of making and sending the orders as well as the inventory costs will still remain. On the other hand, there is no role for the transfer price C in this case.

As a result, the profits of the integrated monopoly are:

$$\pi(D,n,q) = R(D) - C_v D - h_0 \frac{(n-1)q}{2} - k_0 \frac{D}{nq} - h\frac{q}{2} - (k+A)\frac{D}{q} \tag{1}$$

The relevant variables for the integrated firm are then the annual demand (D), the batch size (q) and the factor defining the batch when the firm places orders with its providers (n).

Following Viswanathan and Wang (2003), for a given demand level D and factor n, it is possible to obtain the batch size by solving:

$$\frac{d\pi}{dq} = \frac{k_0 D}{nq^2} - \frac{(n-1)h_0}{2} - \frac{h}{2} + (k+A)\frac{D}{q^2} = 0$$

And as a result,

$$q^*(D,n) = \sqrt{\frac{2D\left[k + A + (k_0/n)\right]}{h - h_0 + h_0 n}} \tag{2}$$

Replacing q^* in the integrated profits and defining an auxiliary variable L as:

$$L = \left[K + A + \frac{k_0}{n} \right] (h - h_0 + h_0 n)$$

It is possible to write the profits as:

$$\pi(D, n) = R(D) - C_v D - \sqrt{2LD}$$

It is then clear that, in order to maximize $\pi(D, n)$ we should minimize L. Defining $\lfloor x \rfloor$ as the largest integer lower or equal to x, a closed form expression for n^* is given by:

$$n^* = \left\lfloor \frac{1 + \sqrt{1 + 4(h - h_0)k_0/h_0(k + A)}}{2} \right\rfloor \quad (3)$$

Finally, the price decision, or equivalently the quantity decision D, is obtained from:

$$\frac{d\pi}{dD} = R'(D) - C_v - \sqrt{L/2D} = 0 \quad (4)$$

2.3 The model under vertical control tools

In this section we consider again the non-integrated structure, but we look for a solution that reproduces the integrated optimum given by (2), (3) and (4). In the literature we can found different approaches to reach the goal (see Viswanathan and Wang, 2003), but here we explore the use of vertical control tools, in particular, a non-linear pricing scheme.

The idea is to pass wholesaler costs directly to the retailer, such that the retailer confronts the same problem as the integrated firm.[1] The benefits generated by the retailer are then extracted by the wholesaler by the use of a fixed component in the tariff. As a result, the wholesaler obtains the integrated profits in avery simple way and without vertical integration.

In the case under analysis, in order to transfer all his costs to the retailer, the wholesaler should establish the following tariff:

$$T(D, q) = F + C_v D + A\frac{D}{q} + k_0 \frac{D}{n^* q} + \frac{h_0 q(n^* - 1)}{2}$$

where n^* is obtained from the solution of the integrated monopoly, since the wholesaler is aware of the integrated results under complete information.

It is worthy to note that the wholesaler proposes a tariff where the variable part contains his marginal cost for each unit sold during the year, but also the inventory costs, the costs to satisfy a batch request and his own batch request costs.

[1] Viswanathan and Wang (2003) reproduced the integrated monopoly profits with a much more complicated scheme that consists of handing over a contract to the retailer and which simultaneously considers discounts for volume and quantity.

Now the wholesaler's profits will be given by:

$$\pi_v = F + C_v D + A\frac{D}{q} + k_0\frac{D}{n^*q} + \frac{h_0 q(n^*-1)}{2} - C_v D - A\frac{D}{q} - k_0\frac{D}{n^*q} - \frac{h_0 q(n^*-1)}{2}$$

i.e.

$$\pi_v = F$$

On the other hand, the retailer's profits are given by:

$$\pi_r(D,q) = R(D) - T(D,q) - h\frac{q}{2} - k\frac{D}{q}$$

and replacing $T(D,q)$ we got:

$$\pi_r(D,q) = R(D) - F - C_v D - \left(A + \frac{k_0}{n^*}\right)\frac{D}{q} - \frac{h_0(n^*-1)q}{2} - h\frac{q}{2} - k\frac{D}{q} \tag{5}$$

Accordingly, the retailer should maximize expression (5) in D and q. It is easy to see that this expression coincides with the objective function of the integrated firm when $n = n^*$. For example,

$$\frac{d\pi_r(D,q)}{dq} = \left(A + \frac{k_0}{n^*} + k\right)\frac{D}{q^2} - \frac{h_0(n^*-1)+h}{2} = 0$$

Therefore:

$$q^* = \sqrt{\frac{2D(k + A + (k_0/n^*))}{h - h_0 + h_0 n^*}}$$

This coincides with the batch size of the integrated firm (see eq. 2) when $n = n^*$. Finally, replacing q^* and n^* in (5) we got:

$$\pi_r(D,q) = R(D) - F - C_v D - \sqrt{2DL^*}$$

where:

$$L^* = \left[K + A + \frac{k_0}{n^*}\right](h - h_0 + h_0 n^*)$$

And then, the optimal D is obtained from:

$$\frac{d\pi_r(D,q)}{dD} = R'(D) - C_v - \sqrt{L^*/2D} = 0$$

As a result, the retailer solves the same problem as the integrated firm, but the rents are easily extracted for the wholesaler by defining F such that $\pi_r(D^*, q^*) = 0$, i.e.,

$$F = R(D^*) - C_v D^* - \sqrt{2D^* L^*}$$

In sum, we have shown that a very easy way to reproduce the rents of the integrated monopoly is by using a conveniently defined non-linear pricing squeme. There is no need for complicated numerical approximations founded in the literature. However, it is fair to be worried about the feasibility to extend this simple approximation to more complex scenarios. In fact we show in the next section that it is posible to do it, but eventually dealing with scenarios of strategic interaction.

3 Extension of the model: m retailers

In this section we consider an extension of the basic model where the wholesaler interacts with m retailers (with $m \geq 2$), in m different regions, each one of them exerting monopoly power at its location.

The wholesaler sells the product at a unitary price C per unit and faces a cost A to satisfy an order from any of its retailers. As before, there is a direct cost of $C_v D$ by D units bought to its providers, an order cost of k_0 and an inventory cost f h_0 per unit held per year. Retailer i faces an inventory cost h_i, for each unit hold per year and an order cost k_i. The retailer i faces a price-sensitive deterministic demand $D_i(p_i)$ in each period and must choose a linear pricing. The income for retailer i can be written as $R_i(D_i) = \psi_i(D_i)D_i$, where $\psi_i(\cdot)$ is the inverse demand function ($p_i = \psi_i(D_i)$).

The batch size of retailer i is denoted by q_i and we assume that the batch size of wholesaler is given by $q_v = nq \equiv n\sum_{j=1}^{m} q_j$ with n integer. Given that retailers are not placing orders at the same time, what we are assuming is that m is sufficiently high. The situation is summarized in Figure 3.

Following the same approach used in previous section, it is easy to see that profits for retailer i are given by:

$$\pi_i(D_i, q_i) = R_i(D_i) - CD_i - h_i \frac{q_i}{2} - k_i \frac{D_i}{q_i} \qquad i = 1...m \tag{6}$$

while wholesaler's profits can be written as:

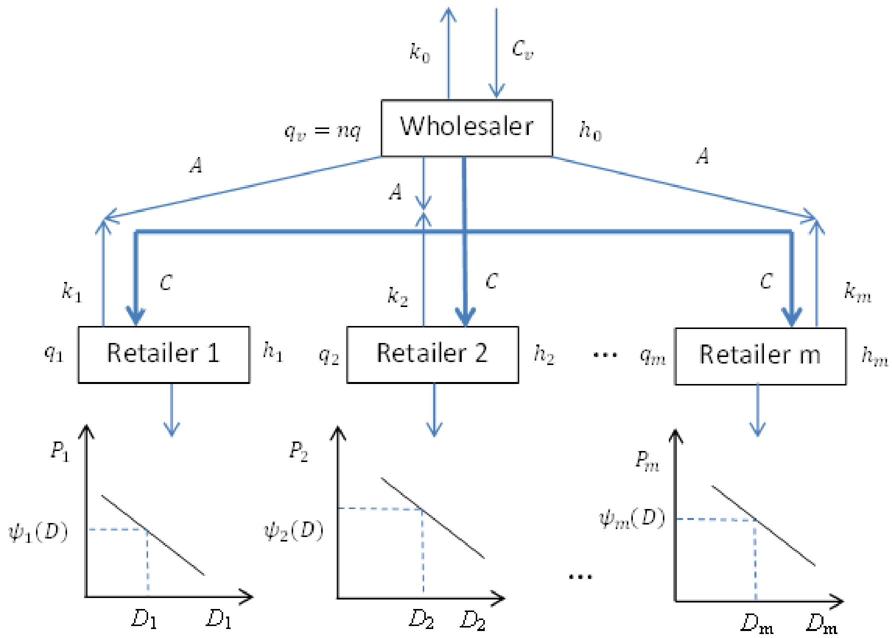

Figure 3: Diagram for the case with "m" retailers

$$\pi_v(C,n) = C\sum_{i=1}^{m} D_i - C_v\sum_{i=1}^{m} D_i - \frac{h_0(n-1)}{2}\sum_{i=1}^{m} q_i - A\sum_{i=1}^{m}\frac{D_i}{q_i} - k_0\frac{\sum_{i=1}^{m} D_i}{n\sum_{i=1}^{m} q_i} \tag{7}$$

Accordingly, the profits for the integrated firm are obtained from adding all the profit functions in (6) and (7):

$$\pi(D_1,...,D_m,n,q_1,...q_m) = \sum_{i=1}^{m}\pi_i(D_i,q_i) + \pi_v(C,n)$$

the result does not depend on C and it is given by:

$$\begin{aligned} \pi(D_1,...,D_m,n,q_1,...q_m) \quad = \quad & \sum_{i=1}^{m} R_i(D_i) - C_v\sum_{i=1}^{m} D_i - \frac{h_0(n-1)}{2}\sum_{i=1}^{m} q_i \\ & -k_0\frac{\sum_{i=1}^{m} D_i}{n\sum_{i=1}^{m} q_i} - A\sum_{i=1}^{n}\frac{D_i}{q_i} - \sum_{i=1}^{m} h_i\frac{q_i}{2} - \sum_{i=1}^{m} k_i\frac{D_i}{q_i} \end{aligned} \tag{8}$$

Leading to a set of First Order Conditions (FOC) that characterize the optimum $(D_1^*, ..., D_m^*, n^*, q_1^*, ...q_m^*)$

3.1 The model under vertical control tools

In this section we look for a decentralized solution that reproduces the integrated optimum using a non-linear pricing scheme. We define:

$$T_i(D_1, ..., D_m, q_1, ...q_m) = F_i + C_v D_i + \frac{h_0(n^* - 1)}{2} q_i + A\frac{D_i}{q_i} + k_0 \frac{\sum_{j=1}^{m} D_j}{n^* \sum_{j=1}^{m} q_j}$$

Where n^* is obtained by the wholesaler solving the integrated case,[2] which is posible because we are under complete information. Profits for retailer i are given now by:

$$\pi_i(D_1, ..., D_m, q_1, ...q_m) = R_i(D_i) - T_i(D_1, ..., D_m, q_1, ...q_m) - h_i\frac{q_i}{2} - k_i\frac{D_i}{q_i} \qquad i = 1...m$$

and it is easy to check that, for given n^*, retailer i will get the same first order conditions for (D_i, q_i) with $i = 1...m$. It is important to note that retailers profits are not independent, in fact they are playing a static game of complete information, so the solution we got is a Nash equilibrium of the game $(D_1^*, q_1^*; D_2^*, q_2^*; ...; D_m^*, q_m^*)$. The equilibrium level for each variable coincides with the optimal level obtained for the integrated monopoly. The reason why the profit functions are not independent is because the wholesaler wants that retailer i internalizes the total effect that his/her decisions have on the ordering costs. It is important to mention too that the set of variables retailer i has to choose is the same as in the basic case with a linear tariff. What has change with the use of a non-linear tariff is that wholesaler has been able to align retailers' incentives with their own.

Finally, F_i is selected so that retailers' profits are zero, so:[3]

$$F_i = R_i(D_i^*) - C_v D_i^* - \frac{h_0(n^* - 1)}{2} q_i^* - A\frac{D_i^*}{q_i^*} - k_0 \frac{\sum_{j=1}^{m} D_j^*}{n^* \sum_{j=1}^{m} q_j^*} - h_i\frac{q_i^*}{2} - k_i\frac{D_i^*}{q_i^*} \qquad i = 1...m$$

[2] In the Appendix we show how n^* must be obtained.

[3] Note that F_i must be a constant.

so in equilibrium the wholesaler obtains:

$$\pi_v = \sum_{i=1}^{m} T_i(D_1^*, ..., D_m^*, q_1^*, ...q_m^*) - \sum_{i=1}^{m} C_v D_i^* - \frac{h_0(n^*-1)}{2}\sum_{i=1}^{m} q_i^* - \sum_{i=1}^{m} A\frac{D_i^*}{q_i^*} - k_0\frac{\sum_{i=1}^{m} D_i^*}{n^*\sum_{i=1}^{m} q_i^*}$$

and replacing $T_i(D_1^*, ..., D_m^*, q_1^*, ...q_m^*)$ we got:

$$\begin{aligned}\pi_v &= \sum_{i=1}^{m} R_i(D_i^*) - \sum_{i=1}^{m} h_i\frac{q_i^*}{2} - \sum_{i=1}^{m} k_i\frac{D_i^*}{q_i^*} - \sum_{i=1}^{m} C_v D_i^* \\ &\quad - \frac{h_0(n^*-1)}{2}\sum_{i=1}^{m} q_i^* - \sum_{i=1}^{m} A\frac{D_i^*}{q_i^*} - k_0\frac{\sum_{i=1}^{m} D_i^*}{n^*\sum_{i=1}^{m} q_i^*}\end{aligned} \quad (9)$$

It is clear that, in equilibrium, the profits in equation (9) coincide with those obtained in equation (7) by the integrated monopoly at the optimum.

4 Conclusions

In this paper we study agents' decisions in distribution channels where final demands are price sensitive. Following the literature, we consider the integrated case as the relevant reference to identify the maximum achievable profit. However, we depart from the literature in the tools used to reproduce such scenario.

We showed the wholesaler can use non-linear tariffs to align retailer incentives in order to get the reference profits given by the integrated case. It is shown that in fact integrated profits are obtained as the optimal solution in a one wholesaler - one retailer case, and as a Nash equilibrium solution in a one wholesaler - m retailers case. The use of non-linear tariffs non only succeed in reproduce the integrated case, but it is a remarkably simple way to do it in relation to other approaches suggested in the literature.

Some interesting extensions of this work are given by scenarios where price discrimination is permitted in final markets. In such a case our approach also should reproduce the optimal result. More interesting is the case under incomplete information, where some informational rents should be left to retailers in order to align incentives.

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6 Appendix

In this appendix we obtain n^* for the m retailers case.